Math 135: General Course Outline

Catalog Description

135. Ordinary Differential Equations. (4) Lecture, three hours; discussion, one hour. Requisites: courses 33A, 33B. Selected topics in differential equations. Laplace transforms, existence and uniqueness theorems, Fourier series, separation of variable solutions to partial differential equations, Sturm-Liouville theory, calculus of variations, two point boundary value problems, Green's functions. P/NP or letter grading.

Textbook

G. Simmons, *Differential Equations with Applications and Historical Notes,* 2nd Ed., McGraw-Hill.

Schedule of Lectures

Lecture	Sections	Topics
1	.	General course overview.
2	17, 18	Review of solution methods and properties of solutions for linear constant coefficient equations.
3	48, 50, 51	Laplace transform. Forward transform, inverse transform. Examples of transform pairs.
4	48, 50, 51	The Laplace transform of a differential equation. The use of Laplace transforms for the solution of initial value problems.
5	48, 50, 51	Computation of the inverse Laplace transform. Partial fraction expansions revisited[1].
6	49	Existence and uniqueness of Laplace transforms. Sectionally continuous functions. Exponentially bounded functions.
7	52, 53	Proof of the convolution theorem. The Heavyside expansion theorem[2].
8	52, 53	The Heavyside function and Dirac distribution. Unit impulse response functions. Use of the unit impulse response function[3].
9	68, 69	Existence and uniqueness theory. Examples of differential equations without unique solutions or global solutions. Lipschitz condition; determination of Lipschitz constants.
10	68, 69	Statement of a global existence and uniqueness theorem -- when f(x,y) is Lipschitz in [a,b] x [-∞, ∞][4]. Examples of the application of the existence and uniqueness theorem.
11	68, 69	Outline of the proof of existence and uniqueness theorem. Proof preliminaries; max norm, uniform convergence, Weierstrauss M-test. Equivalence of the differential equation to an integral equation[5].
12	68, 69	Picard iteration. Proof of existence and uniqueness.
13	68, 69	Local existence and uniqueness theorems. Applications of local existence and uniqueness theorems.
14	.	Midterm
15	33	Periodic functions and Fourier series. The inadequacy of power series approximations for periodic functions. Fourier series coefficient formulas. Examples of Fourier series.
16	35, 36	Derivation of Fourier series coefficient formulas. Fourier series for periodic functions over arbitrary intervals.
17	37	Function inner products. Orthogonal functions. Derivation of Fourier series coefficient formulas using inner products.

18	34, 38	Convergence theorems for Fourier series: Pointwise convergence.
19	34, 38	Convergence theorems for Fourier series: L^2 convergence (Mean convergence).
20	40	Eigenvalues and Eigenfunctions of two point boundary value problems.
21	41	Separation of variables solution to one dimensional heat equation.
22	42	Separation of variables solution to Laplace's equation in a disk.
23	43	Sturm-Liouville problems.
24	43	Leeway
25	65, 66, 67	Calculus of Variations: Introduction.
26	65, 66, 67	Euler's differential equation for an extremal.
27	65, 66, 67	Isoperimetric problems.
28	.	Review

Comments

Footnotes

1. The book does not include a review of partial fractions. Most calculus textbooks provide a suitable discussion of the technique.

2. The book only states a limited form of the Heavyside expansion theorem in problem 5 of section 53. The more general statement can be found in standard texts devoted to Laplace transforms.

3. The book provides a limited description of the use of the unit-step function and unit impulse functions. A better treatment can be found in Redheffer's book *Differential Equations.*

4. The proof of Theorem B is easier than Theorem A (the local existence theorem) since one doesn't have to worry about the Picard iterates leaving the domain where f(x,y) is Lipschitz. Thus, discussing and proving Theorem B before Theorem A is recommended.

5. The book glosses over some of the mathematical details required by the convergence proofs so one must supplement the material in the text as needed.

Additional Notes

An energetic instructor may want to cover two point boundary value problems and Green's functions in more depth instead of spending the last three lectures on the calculus of variations. Alternately, one could replace the lectures on the calculus of variations with lectures on regular perturbation theory. A reference for this latter topic is Bender and Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, Chapter 7.

Outline update: C. Anderson, 5/05

NOTE: While this outline includes only one midterm, it is strongly recommended that the instructor considers giving two. It is difficult to schedule a second midterm late in the quarter if it was not announced at the beginning of the course.

For more information, please contact Student Services, ugrad@math.ucla.edu.