Math 134: General Course Outline

Catalog Description

134. Linear and Nonlinear Systems of Differential Equations. (4) (Formerly numbered 135A.) Lecture, three hours; discussion, one hour. Requisites: courses 33B, 115A. Dynamical systems analysis of nonlinear systems of differential equations. One- and two- dimensional flows. Fixed points, limit cycles, and stability analysis. Bifurcations and normal forms. Elementary geometrical and topological results. Applications to problems in biology, chemistry, physics, and other fields. P/NP or letter grading.

Textbook

S. Strogatz, *Nonlinear Dynamics and Chaos*, Perseus Books Group.
J. Crawford, *Introduction to Bifurcation Theory*, Reviews of Modern Physics, vol. 63. (Recommended supplement). (http://prola.aps.org/abstract/RMP/v63/i4/p991_1)

Schedule of Lectures

Lecture	Sections	Topics
1	.	General course overview.
2	1.0 - 1.3	Definition of dynamical systems. Discussion of importance and difficulty of nonlinear systems. Examples of applications giving rise to nonlinear models.
3	2.0 - 2.3	Elementary one-dimensional flows. Flows on the line, fixed points, and stability. Application to population dynamics. Discussion of how geometric "dynamical systems" approach is different from approach in Math 33.
4	2.4 - 2.6	"Advanced" one-dimensional flows. Linear stability analysis (with numerous examples), existence and uniqueness, impossibility of oscillations.
5	2.6 - 2.7	Potentials. Introduction to the idea of numerical solutions of nonlinear equations, including discussion of basic methods, software tools (Matlab, Maple, Mathematica, DSTool, xppaut, etc.). Advertisement for Math 151A/B.
6	3.0 - 3.1	Introduction to bifurcations, saddle-node bifurcation. Physical relevance of bifurcations, introduction to bifurcation diagrams, notion of normal forms. For saddle-node bifurcation, incorporate treatment in Crawford.
7	3.2 - 3.3	Transcritical bifurcation. Incorporate treatment in Crawford. Extended example on laser threshold.
8	3.4 - 3.5	Pitchfork bifurcation. Incorporate treatment in Crawford. Extended example on overdamped bead on rotating hoop.
9	3.5	Dimensional analysis. Basic technique. Relate to overdamped bead example.
10	3.6 - 3.7	Imperfect bifurcations. Basic theory and bifurcation diagrams. Insect outbreak model, time permitting.
11	4.0 - 4.3	Flows on the circle. Definition, beating, nonuniform oscillators, ghosts and bottlenecks.
12	4.4 - 4.6	Oscillator examples. Instructor should choose one or two of the examples (overdamped pendulum, fireflies, superconducting Josephson junctions) to cover in depth.
13	5.0 - 5.1	Introduction to two-dimensional linear systems. Motivating examples, mathematical set-up, definitions, different types of stability. Phase portraits, stable and unstable eigenspaces.

14	5.2	Classification of linear systems. Eigenvalues, eigenvectors. Characteristic equation, trace and determinant. Different types of fixed points. (Suggestion: cover example material in Section 5.3 and related problems on homework.)
15	.	Midterm
16	6.0 - 6.2	Introduction to two-dimensional nonlinear systems. Phase portraits and null-clines. Existence, uniqueness, and strong topological consequences for two-dimensions.
17	6.3	Equiliria and stability. Fixed points and linearization. Effect of nonlinear terms. Hyperbolicity and the Hartman-Grobman theorem.
18	6.5 - 6.6	Special nonlinear systems. Conservative and reversible systems. Heteroclinic and homoclinic orbits.
19	6.7	Extended application of nonlinear phase plane analysis to classic pendulum problem without restricting to small-angle regime. (Alternatively: another application of the instructor's choice.)
20	6.8	Index theory. Discussion of local versus global methods. Definition and useful properties of the index, with examples.
21	7.0 - 7.1	Introduction to limit cycles. Definition. Polar coordinates. Van der Pol oscillator and other examples.
22	7.2	Ruling out limit cycles. Gradient systems, Liapunov functions, and Dulac's criterion, with examples.
23	7.3	Proving existence of closed orbits. Poincare-Bendixson theorem, trapping regions. Examples. Impossibility of chaos in the phase plane.
24	8.0 - 8.1	Bifurcations in two (and more) dimensions. Revisitation of saddle-node, transcritical, and pitchfork bifurcations, with examples.
25	8.2 - 8.3	Hopf bifurcation. Definition. Supercritical, subcritical, and degenerate types. Application to oscillating chemical reactions if time permits.
26	8.4	Global bifurcations of cycles. Saddle-node, infinite-period, and homoclinic bifurcations. Scaling laws for amplitude and period of limit cycle.
27	.	leeway
28	.	Review

Comments

For those instructors wishing to incorporate a final project, lectures 9 and 10 can be skipped and the last four lectures can be used for final project poster presentations.

If time is available for more lectures than those outlined, additional lectures could cover section 7.6 (on weakly nonlinear oscillations and perturbation theory) or selected sections from chapter 9 (on chaos and the Lorenz equations).

Outline update: C. Topaz, 4/04, updated, 3/05

NOTE: While this outline includes only one midterm, it is strongly recommended that the instructor considers giving two. It is difficult to schedule a second midterm late in the quarter if it was not announced at the beginning of the course.

For more information, please contact Student Services, ugrad@math.ucla.edu.