Math 133: General Course Outline

Catalog Description

133. Introduction to Fourier Analysis. (4) Lecture, three hours; discussion, one hour. Requisites: courses 33A, 33B, 131A. Fourier series, Fourier transform in one and several variables, finite Fourier transform. Applications, in particular, to solving differential equations. Fourier inversion formula, Plancherel theorem, convergence of Fourier series, convolution. P/NP or letter grading.

Textbook

E. Stein and R. Shakarchi, *Fourier Analysis: An Introduction (Princeton Lectures in Analysis, Volume 1)*, Princeton University Press.

Reviews & Exams

This syllabus is based on a single midterm; instructors who wish to give a second midterm may adjust the syllabus appropriately, or give the second midterm in section. The lecturer may also wish to expand the applications components (lectures 11-12, 22-24, 26-28) or move them earlier in the course.

Schedule of Lectures

Lecture	Topics
1	Review: Complex numbers (esp. Euler's formula); periodic functions; functions on an interval; functions on a circle; continuous functions; continuously differentiable functions; Riemann integrable functions (or at least piecewise continuous functions). Note that Lebesque integration and L^2 are not covered rigorously in this course.
2	Does every function have a Fourier series? Formal computation of Fourier coefficients. Inversion formula for trigonometric polynomials. Examples of Fourier series (esp. Dirichlet kernel).
3	Review of convergence, uniform convergence. Do Fourier series converge back to the original function? Injectivity of the Fourier transform for continuous functions.
4	Uniform convergence for absolutely summable Fourier coefficients. Relationship between differentiation and the Fourier transform. Uniform convergence for C^2 functions. (Optional) Some foreshadowing of future convergence results.
5	Convolutions of continuous periodic functions: examples and basic properties. Connections with Fourier coefficients. Connection between partial sums and the Dirichlet kernel.
6	Convolutions of integrable periodic functions: approximation of integrable functions by continuous ones. Approximation via convolution by good kernels.
7	Badness of the Dirichlet kernel; Gibbs' phenomenon. Cesaro means; Fejer kernel. Fejer's theorem. Uniform approximation of continuous functions by trigonometric polynomials.
8	Leeway
9	Review of vector spaces, inner product spaces, orthonormal sets, Cauchy-Schwarz inequality, Pythagoras's theorem. Orthonormality of the Fourier basis. Bessel's inequality. Best mean-square approximation by trigonometric polynomials.
10	Mean-square convergence of Fourier series for continuous functions. Mean-square convergence of Fourier series for Riemann-integrable functions. Plancherel's theorem, Parseval's theorem. Riemann-Lebesque lemma.

11-12	Applications and further properties of Fourier series, at instructor's discretion. Some suggestions: Summation of 1/n^2; local convergence of Fourier series at smooth points; smoothness of a function versus decay of Fourier coefficients; a continuous function with divergent Fourier series; comparison of sine and cosine Fourier series with exponential Fourier series; isoperimetic inequality; uniform distribution of multiples of an irrational (Monte Carlo integration); a continuous, nowhere differentiable function.
13	Leeway/review
14	Midterm.
15	From Fourier series to Fourier integrals - an informal discussion. Review of improper integrals. Functions of moderate decrease. Functions of rapid decrease. Schwartz functions. Definition of the Fourier transform.
16	Basic algebraic properties of the Fourier transform. Preservation of the Schwartz space.
17	Fourier transform of Gaussians. Gaussians as good kernels.
18	Multiplication formula. Fourier inversion formula. Bijectivity on Schwartz space.
19	Fourier transform and convolutions. Plancherel's theorem. Extension to functions of moderate decrease.
20-21	Integration on R^d; Fourier transform on R^d; key properties.
22-24	Applications to PDE: heat equation; Laplace's equation. (Optional) The wave equation (in 1D or higher dimensions).
25	Z_N. The finite Fourier transform; key properties.
26-28	Applications and further properties of Fourier transforms, at instructor's discretion. Some suggestions: The fast Fourier transform; fast multiplication; Heisenberg uncertainty principle; Comparison of Fourier and Laplace transforms; The Fourier-Bessel transform for radial functions; Poisson summation formula; band-limited functions and the Shannon sampling theorem; linear transformations and the Fourier transform on R^n; the Dirac delta function.
29	Leeway/review.

For more information, please contact Student Services, ugrad@math.ucla.edu.