

Math 123: Foundations Of Geometry

- [Math 123: General Course Outline & Catalog Description](#)

Catalog Description

123. Foundations of Geometry. Lecture, three hours; discussion, one hour. Prerequisite: course 115A. Axioms and models, Euclidean geometry, Hilbert axioms, neutral (absolute) geometry, hyperbolic geometry, Poincare model, independence of parallel postulate.

General Information

The purpose of Math 123 is to study the axiom sets and models for various geometries, with particular attention paid to Euclid's parallel postulate and to models for geometries that violate the parallel postulate (noneuclidean geometries). The course is particularly useful for prospective secondary school teachers, in that it illustrates how a mathematical structure can be built upon an axiom system, and it puts in perspective the euclidean geometry that is traditionally studied in the schools.

The system of geometry laid down by Euclid around 300 BC in his treatise *The Elements* was based on five postulates, or assumptions, of a geometric nature:

- (1) Any two points can be joined by a (straight) line.
- (2) Any segment can be extended continuously in a (straight) line.
- (3) Given any point and distance, there is a circle centered at the point with radius equal to the distance.
- (4) All right angles are equal to each other.
- (5) Given a line and a point not on the line, there exists one and only one line passing through the point and parallel to the given line.

Euclid's statement of the fifth (parallel) postulate is actually more complicated than the one given above. It seemed as if the parallel postulate should be a consequence of the other postulates, that is, it should be a theorem rather than a postulate. For more than 2000 years mathematicians tried unsuccessfully to deduce the parallel postulate from the other four. In the nineteenth century mathematicians began to consider the possibility that there might be geometries for which the parallel postulate fails, and they began to study the properties of such geometries. At long last some models of geometries for which the parallel postulate fails were discovered. One such geometry is the hyperbolic geometry of the unit disk in the complex plane, in which the "lines" are arcs of circles perpendicular to the unit circle. One of the goals of Math 123 is to study noneuclidean geometries, and in particular to study the hyperbolic geometry of the unit disk.

Math 123 is a flexible course, and it is taught quite differently by different instructors. It is offered once a year, usually Winter Quarter. The class size is capped at about 40, and enrollment may be limited.