Math 121: Introduction to Topology

- Math 121: General Course Outline & Catalog Description

Topology is the study of the properties of spaces (such as surfaces, or solids) that are invariant under homeomorphisms (such as stretchings). One striking theorem in topology is that any compact orientable two-dimensional surface is topologically a sphere with a certain number of handles attached. The number of handles completely characterizes the topological type of the surface. This leads to the adage that a topologist is a person who cannot tell the difference between a teacup and a doughnut. Topologically speaking, each is a sphere with one handle, and each can be continuously deformed to the other.

While topology is classified under geometry, the language of topology is fundamental to analysis. Many of the issues addressed by topology, such as compactness of spaces and continuity of functions, are treated in a simpler setting in the analysis courses 131AB.

One method for studying topological spaces is to assign algebraic objects, such as groups or vector spaces, to a topological space. One such object is the "fundamental group" of a topological space, which measures in some sense the number of holes in the space. Thus topology interacts also with algebra, leading to a branch of mathematics called "algebraic topology."

Math 121 is a flexible course, and the selection of topics might be organized quite differently by different instructors. The subject matter for a standard syllabus breaks into three parts.

The first part treats metric spaces, which are closest to the intuition and to the development presented in 131AB. The fundamental concepts are completeness, compactness, continuity, and uniform continuity. The principal theorems are the Baire category theorem, the characterization of compact metric spaces, the theorem that continuous functions on a compact space are uniformly continuous, and the contraction mapping principle, which is perhaps the most important and useful tool in analysis.

The second part of the standard course covers point-set topology. Topological spaces are introduced, along with the separation axioms and various notions as compactness, local compactness, connectedness, and path connectedness. Product and quotient spaces are defined. The most important theorem in point-set topology is Tychonoff's theorem that the product of a family of compact topological spaces is compact.

The third part of the standard course consists of an elementary introduction to algebraic topology. The fundamental group is introduced, and covering spaces are used to compute it for some special spaces. Some simple applications of the algebraic invariants are given.

Math 121 is offered once each year, usually in the Spring Quarter. Course enrollments run between 10 and 35.