MATH 285G : Perelman's proof of the Poincaré conjecture

- **Course description**: The course will cover as much of Perelman's proof as possible. Specific topics include: Existence theory for Ricci flow, finite time blowup in the simply connected case, Bishop-Cheeger-Gromov comparison theory, Perelman reduced length and reduced volume and applications to non-collapsing, Perelman compactness theorem, structure and asymptotics of ancient κ-non-collapsing solutions, analysis of horns and necks, surgery.

Announcements:

- The first class will be on **Wednesday**, April 2 (I will be in Washington, DC on Monday March 31).

- **Instructor**: Terence Tao, tao@math.ucla.edu, x64844, MS 6183
- **Lecture:** MWF 4-4:50, MS5137
- **Quiz section:** None
- **Office Hours**: M 2-3
- **Textbook**: I will use a number of sources, including Morgan-Tian's "Ricci flow and the Poincaré conjecture", Chow's "Ricci flow: an introduction", Kleiner-Lott's "Notes on Perelman's papers", and various papers, including of course Perelman's. I will post lecture notes on my blog site.
- **Prerequisite**: The Math 234 class (Ricci flow) from last winter is highly recommended. I will put some basic foundational material on PDE and Riemannian geometry in my lecture notes, but if you are not familiar already with Ricci flow, I would expect that one would need a significant amount of self-study in these topics in order to keep up with the course.
- **Grading**: Grading is based on attendance.
- **Reading Assignment**: Lecture notes will be provided on my blog site. Students are encouraged to comment on these posts.
- **Homework**: There is no homework for this course.

Online resources:

- The original papers of Perelman:
  1. The entropy formula for the Ricci flow and its geometric applications
  2. Ricci flow with surgery on three-manifolds

Class 285I, Sprin 2008: Hilbert bimodules and subfactors of finite index
Instructor: Sorin Popa
Meetings: M 2-4, W 2-4 in MS6627.

We begin by reviewing the notion of Hilbert modules over type II_1 factors, and more generally over finite von Neumann (vN) algebras. Then we consider *Hilbert bimodules*, as a generalized notion of symmetries for II_1 factors and vN algebras, and introduce an important tool for studying such objects, called the *the basic construction*. We'll discuss at length several applications to this construction, such as:

- A non-commutative treatement of *relative compactness* and *relative weak mixing* for actions of groups on finite vN algebras.
- Providing several equivalent definitions of finite index for subalgebras;
- Proving criteria for unitary conjugacy between subalgebras in a factor (*intertwining subalgebras* techniques).
- Defining relative amenability, relative property (T), etc.

We then prove Jones' theorem showing that the dimension of Hilbert bimodules can only take values in the set $\{4cos^2\pi/n \mid n \geq 3\} \cup [0, \infty]$. We introduce the *standard invariant* of a subfactor, and prove several axiomatizations of these objects as "abstract" objects. Finally, we will explain how subfactor techniques can be used to approach problems in *non-commutative ergodic* theory, i.e., in the study and classification of actions of groups on arbitrary (not necessarily abelian) finite vN algebras.

All registered students in the class who will attend regularly will get A, but they are expected to present some assigned material in the 290 Student Seminar, Mo 4-5, in MS 6229. The first lecture will be Wednesday April 2'nd, and instead of 2-4pm, we will have it ~~3-5pm~~ 4-6 PM that day. (as there will be no Functional Analysis Seminar 4-5 until Wed April 16).