Mathematics 275C - Spring 2008

Stochastic Processes

- **Time and place:** MWF at 10 in MS5233
- **Instructor:** Thomas M. Liggett
- **Office hours:** MWF 11-12 in MS 7919
- **Text:** Probability: Theory and Examples, third edition by R. Durrett. We will cover most of Chapter 7 + additional material that does not appear in the text.
- **Prerequisite:** Mathematics 275B (especially discrete time martingales) or equivalent.
- **Grading:** Grades will be based on homework. There will be no exams.

Topics

- **Brownian motion and applications.** Brownian motion is surely the most important continuous time stochastic process. It is, for example, the main building block for the theory of stochastic calculus and mathematical finance, which is the subject of Math 285K in Fall of 2008 (taught by R. Schonmann). Among the topics we will discuss are: (a) its definition and construction, (b) path properties, (c) the strong Markov property and its uses in performing explicit calculations, and (d) the Skorokhod representation, which permits reduction of problems involving iid random variables to Brownian motion problems. Brownian motion is the main topic of the course.
- **More general continuous time Markov processes.** Using Brownian motion and pure jump processes as motivating examples, we will discuss the basics of more general Markov process theory, including semigroups and generators. Other examples that may be discussed are Levy (infinitely divisible) processes and the special case of stable processes.
- **Brownian motion and the Dirichlet problem.** The Dirichlet problem asks for harmonic functions on a domain D in Euclidean space with prescribed boundary conditions. The approach to this problem based on Brownian motion has a number of advantages over purely analytic approaches, including (a) treatment of domains that are unbounded and/or do not have smooth boundary, and (b) a probabilistic interpretation for the solution(s).