Math 207C

Elliptic Curves

Elliptic curves lie at the heart of contemporary number theory, and a basic understanding of them is key to graduate work in the field. They also arise naturally in a number of other fields including cryptography and combinatorics. This introductory course will define them in a general setting and prove the basic results. Examples include the Mordell-Weil theorem and the computation of the Mordell-Weil group, the Riemann Hypothesis for elliptic curves over finite fields, and the Schoof-Elkies algorithm for computing the group of points, the theory of the non-abelian Galois extensions generated by torsion points, the theory of the height pairing (and applications to sphere packing), and the theory of supersingular curves and recent applications to construction of expander graphs and hash functions in cryptography, the Tate-Shafarevitch group, the theory of reduction mod p, the connection(s) with modular forms, the Shimura-Taniyama Conjecture (now a theorem of Wiles and others) and the Conjecture of Birch and Swinnerton-Dyer.

The course will require no background other than a mastery of the basic grad algebra syllabus and some complex analysis. A parallel introduction to algebraic geometry, taught by Hida this quarter, will give needed background from that area. However, results from that field (such as the Riemann-Roch theorem) will be carefully stated, if not proven in this course.

The course meets Monday and Wednesday, 3-4:15 in MS 7608.

There will be homework assignments and a final study project, with classroom presentation, for evaluation. The subject is sufficiently rich that all students (even those primarily interesting in analysis) should be able to find a topic that interests them.

For further information, please email me. The first lecture will state some of the target theorems and begin the theory of elliptic functions, i.e. we will start the course over C, with doubly periodic meromorphic functions in the the complex plane.

Don Blasius

Math 223s, Set Theory

Spring Quarter 2008

The class will cover the basics of forcing, a technique introduced by Cohen to prove the independence of the Continuum Hypothesis from the axioms of mathematics.

Recall that two sets A and B have the same cardinality if there is a bijective function $f\colon A \to B$. The Continuum Hypothesis states that every infinite subset of $\mathbb{R}$ has either the same cardinality as $\mathbb{R}$, or the same cardinality as $\mathbb{N}$; there are no cardinalities in between.

It turns out that the Continuum Hypothesis is *independent*, meaning neither provable nor refutable, from the axioms of mathematics. That it cannot be refuted was shown by Gödel (1940's) and that it cannot be proved was shown by Cohen (1960's). Cohen's technique has since been used in proofs of many other independence results. For example Solovay used it to show that the existence of a non-measurable set of reals cannot be proved without the axiom of choice.

The class will cover the basics of the forcing technique, preservation of cardinals in forcing extensions, applications to cardinal arithmetic (including the independence of the CH), iterated forcing, Martin's axiom, and applications of Martin's axiom.

Text: *Set Theory, an Introduction to Independence Proofs* by Kenneth Kunen, particularly Chapters 2, 7, and 8.

Prerequisites: A sharp mind, and plenty of time. Students who have not taken 220C may wish to read Chapter 1 in Kunen ahead of time.

Grading: Grading will be based on homework assignments and short presentations in class.

Itay Neeman
ineeman@math.ucla.edu

March 15, 2007

MATH 246A - Spring 2007

Complex Analysis

MWF 1:00 MS 5117 and Thurs 1:00 MS 5117

Office hours:

John Garnett: MWF 4:00 in MS 7941

William Meyerson: TBA in MS 2961

Texts:

1) L. Ahlfors, Complex Analysis, 3rd. Edition, (0-07-000657-1) (required)

2) D. Sarason, Complex Function Theory, 2nd. Edition, American Mathematical Society, 2007 (0-8218-4428-8) (recommended)

3) T. W. Gamelin, Complex Analysis, (0-387-95069-9) (recommended)

Grades: Homework 30%, final 50%, midterm 20%. There will be four homework assignments of 15 - 20 problems each. You must also present at least one homework problem at the blackboard in quiz section.

Prerequisites: Rigorous advanced calculus: Properties of $\mathbb{R}$, least upper bounds, uniform convergence of sequences of continuous functions ($\Rightarrow$ limit is continuous and Riemann integral of limit is limit of integrals), compact and connected sets in $\mathbb{R}^n$. Also, the ability to write a correct mathematical proof. However, neither Math 245 nor undergraduate complex analysis are required prerequisites.

Material: Most of Chapters 1 - 4 of Ahlfors, except the Elementary Point Set Topology section which will be assumed. The rest of the Ahlfors book will be covered in 246B in Fall 2008.

Homework Assignment 1, due Monday April 14: All from Ahlfors, 3rd. Edition.

p. 6, #1. p. 9, #3, 4, 5. p. 11, #1, 4. p. 15, #2, 4. p. 16. #4, 5. p. 17. #2, 3, 5. p. 20. #1, 2, 4, 5.

Notes: The midterm exam will be May 5. There will a review session May 2. There will be a lectures in Thursday Section on April 3 and April 24.

Mathematics 275C - Spring 2008

Stochastic Processes

- **Time and place:** MWF at 10 in MS5233
- **Instructor:** Thomas M. Liggett
- **Office hours:** MWF 11-12 in MS 7919
- **Text:** Probability: Theory and Examples, third edition by R. Durrett. We will cover most of Chapter 7 + additional material that does not appear in the text.
- **Prerequisite:** Mathematics 275B (especially discrete time martingales) or equivalent.
- **Grading:** Grades will be based on homework. There will be no exams.

Topics

- **Brownian motion and applications.** Brownian motion is surely the most important continuous time stochastic process. It is, for example, the main building block for the theory of stochastic calculus and mathematical finance, which is the subject of Math 285K in Fall of 2008 (taught by R. Schonmann). Among the topics we will discuss are: (a) its definition and construction, (b) path properties, (c) the strong Markov property and its uses in performing explicit calculations, and (d) the Skorokhod representation, which permits reduction of problems involving iid random variables to Brownian motion problems. Brownian motion is the main topic of the course.
- **More general continuous time Markov processes.** Using Brownian motion and pure jump processes as motivating examples, we will discuss the basics of more general Markov process theory, including semigroups and generators. Other examples that may be discussed are Levy (infinitely divisible) processes and the special case of stable processes.
- **Brownian motion and the Dirichlet problem.** The Dirichlet problem asks for harmonic functions on a domain D in Euclidean space with prescribed boundary conditions. The approach to this problem based on Brownian motion has a number of advantages over purely analytic approaches, including (a) treatment of domains that are unbounded and/or do not have smooth boundary, and (b) a probabilistic interpretation for the solution(s).

MATH 285G : Perelman's proof of the Poincaré conjecture

- **Course description**: The course will cover as much of Perelman's proof as possible. Specific topics include: Existence theory for Ricci flow, finite time blowup in the simply connected case, Bishop-Cheeger-Gromov comparison theory, Perelman reduced length and reduced volume and applications to non-collapsing, Perelman compactness theorem, structure and asymptotics of ancient κ-non-collapsing solutions, analysis of horns and necks, surgery.

Announcements:

- The first class will be on **Wednesday**, April 2 (I will be in Washington, DC on Monday March 31).

- **Instructor**: Terence Tao, tao@math.ucla.edu, x64844, MS 6183
- **Lecture:** MWF 4-4:50, MS5137
- **Quiz section:** None
- **Office Hours**: M 2-3
- **Textbook**: I will use a number of sources, including Morgan-Tian's "Ricci flow and the Poincaré conjecture", Chow's "Ricci flow: an introduction", Kleiner-Lott's "Notes on Perelman's papers", and various papers, including of course Perelman's. I will post lecture notes on my blog site.
- **Prerequisite**: The Math 234 class (Ricci flow) from last winter is highly recommended. I will put some basic foundational material on PDE and Riemannian geometry in my lecture notes, but if you are not familiar already with Ricci flow, I would expect that one would need a significant amount of self-study in these topics in order to keep up with the course.
- **Grading**: Grading is based on attendance.
- **Reading Assignment**: Lecture notes will be provided on my blog site. Students are encouraged to comment on these posts.
- **Homework**: There is no homework for this course.

Online resources:

- The original papers of Perelman:

  1. The entropy formula for the Ricci flow and its geometric applications
  2. Ricci flow with surgery on three-manifolds

Class 285I, Sprin 2008: Hilbert bimodules and subfactors of finite index
Instructor: Sorin Popa
Meetings: M 2-4, W 2-4 in MS6627.

We begin by reviewing the notion of Hilbert modules over type II_1 factors, and more generally over finite von Neumann (vN) algebras. Then we consider *Hilbert bimodules*, as a generalized notion of symmetries for II_1 factors and vN algebras, and introduce an important tool for studying such objects, called the *the basic construction*. We'll discuss at length several applications to this construction, such as:

- A non-commutative treatement of *relative compactness* and *relative weak mixing* for actions of groups on finite vN algebras.
- Providing several equivalent definitions of finite index for subalgebras;
- Proving criteria for unitary conjugacy between subalgebras in a factor (*intertwining subalgebras* techniques).
- Defining relative amenability, relative property (T), etc.

We then prove Jones' theorem showing that the dimension of Hilbert bimodules can only take values in the set $\{4cos^2\pi/n \mid n \geq 3\} \cup [0, \infty]$. We introduce the *standard invariant* of a subfactor, and prove several axiomatizations of these objects as "abstract" objects. Finally, we will explain how subfactor techniques can be used to approach problems in *non-commutative ergodic* theory, i.e., in the study and classification of actions of groups on arbitrary (not necessarily abelian) finite vN algebras.

All registered students in the class who will attend regularly will get A, but they are expected to present some assigned material in the 290 Student Seminar, Mo 4-5, in MS 6229. The first lecture will be Wednesday April 2'nd, and instead of 2-4pm, we will have it ~~3-5~~pm 4-6 PM that day. (as there will be no Functional Analysis Seminar 4-5 until Wed April 16).

Math 285 J / Sec 1

Algebraic Methods in Combinatorics

Time and Place:

Monday, Friday 3:00-4.50 P.M., MS 5127. First class - ~~April 2~~ MARCH 31

Instructor:

Benny Sudakov, MS 6921, bsudakov@math.ucla.edu

Course description:

Combinatorics is a fundamental mathematical discipline as well as an essential component of many mathematical areas, and its study has experienced an impressive growth in recent years. While in the past many of the basic combinatorial results were obtained mainly by ingenuity and detailed reasoning, the modern theory has grown out of this early stage and often relies on deep, well-developed tools.

One of the main general techniques that played a crucial role in the development of Combinatorics was the application of algebraic methods. The most fruitful such tool is the dimension argument. Roughly speaking, the method can be described as follows. In order to bound the cardinality of of a discrete structure A one maps its elements to vectors in a linear space, and shows that the set A is mapped to linearly independent vectors. It then follows that the cardinality of A is bounded by the dimension of the corresponding linear space. This simple idea is surprisingly powerful and has many famous applications.

This course provides a gentle introduction to Algebraic methods, illustrated by examples and focusing on basic ideas and connections to other areas. The topics covered in the class will include (but are not limited to):

Basic dimension arguments, Spaces of polynomials and tensor product methods, Eigenvalues of graphs and their application, the Combinatorial Nullstellensatz and the Chevalley-Warning theorem. Applications such as: counterexample to Borsuk's conjecture, chromatic number of the unit distance graph of Euclidean space, explicit constructions of Ramsey graphs, ~~and many others~~. KAKEYA PROBLEM IN FINITE FIELDS AND MANY OTHERS

Math 285J, Section 2, Spring 2008
Seminar Applied Mathematics: Mathematical Models in Image Analysis
Lecture Meeting Time: MWF 3.00PM - 3.50PM, MS 5138.

Instructor: Luminita A. Vese

Course Description:
This seminar is devoted to mathematical models arising in image analysis.

- Theory topics: calculus of variations, energy minimization, duality theory, Euler-Lagrange equations, optimality conditions, functions of bounded variation, functionals with linear growth and with jumps, geometric non-linear partial differential, viscosity solutions, oscillatory functions, Sobolev gradients.

- Applications: image restoration (denoising, deblurring), image decomposition into cartoon and texture, image segmentation and edge detection, snakes, curve evolution, active contours, level set methods.

The lectures will not follow one particular textbook. The topics presented can be found in research papers or graduate textbooks.

Textbook References:

- G. Aubert and P. Kornprobst, Mathematical Problems in Image Processing, (Partial Differential Equations and the Calculus of Variations), Springer, 2002 or 2006.
- Y. Meyer, Oscillating Patterns in Image Processing and Nonlinear Evolution Equations, AMS 2001.
- J.-M. Morel and S. Solimini, Variational Methods in Image Segmentation: With Seven Image Processing Experiments (Progress in Nonlinear Differential Equations and Their Applications), Birkhauser 1994.
- S. Osher and R. Fedkiw, Level Set Methods and Dynamic Implicit Surfaces, Springer-Verlag, 2002.
- J. Sethian, Level Set Methods and Fast Marching Methods : Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science, Cambridge University Press, 1999.
- S. Osher and N. Paragios (Eds), Geometric Level Set Methods in Imaging, Vision, and Graphics, Springer-Verlag Telos, 2003.
- R. Kimmel, Numerical Geometry of Images: Theory, Algorithms, and Applications, Springer-Verlag, 2003.
- L. Ambrosio, N. Fusco, D. Pallara, Functions of Bounded Variation and Free Discontinuity Problems (Oxford Mathematical Monographs), Oxford University Press, 2000.
- F. Andreu-Vaillo, V. Caselles, J.M. Mazon, Parabolic Quasilinear Equations Minimizing Linear Growth Functionals, Birkhauser, 2004.
- Book manuscripts by J.-M. Morel and collaborators:
http://www.cmla.ens-cachan.fr/Membres/morel.html
- T.F. Chan and J. Shen, Image processing and analysis, SIAM 2005
- R. Malladi (Ed.), Geometric Methods in Bio-Medical Image Processing, Springer 2002.
- G. Sapiro, Geometric Partial Differential Equations and Image Processing, Cambridge University Press, January 2001.

290E / lec 2
Chern Classes

Instructor: Peter Petersen, MS 6913.
Time: Tu 10am-Noon.
Place: ~~TBA~~ MS 6118
Texts: Milnor-Stasheff, Characteristic Classes,
Additional texts: Hirzebruch, Topological methods in Alegrabic geometry, and Notes by Petersen from website.
Contents: We'll cover the parts of Milnor that avoid Stiefel-Whitney classes and Steenrod squares. Specifically we'll start on page 95, and skip whatever is necessary as we move along in the text.