

Algebraic Methods in Combinatorics, MATH 218B

Time and Place: Monday, Wednesday 2:00-3.50 P.M., MS 5233.
First class: Wednesday, January 7
Instructor: Benny Sudakov, MS 7911, bsudakov@math.ucla.edu

Course description:

Combinatorics is a fundamental mathematical discipline as well as an essential component of many mathematical areas, and its study has experienced an impressive growth in recent years. While in the past many of the basic combinatorial results were obtained mainly by ingenuity and detailed reasoning, the modern theory has grown out of this early stage and often relies on deep, well-developed tools.

One of the main general techniques that played a crucial role in the development of Combinatorics was the application of algebraic methods. The most fruitful such tool is the dimension argument. Roughly speaking, the method can be described as follows. In order to bound the cardinality of a discrete structure A one maps its elements to vectors in a linear space, and shows that the set A is mapped to linearly independent vectors. It then follows that the cardinality of A is bounded by the dimension of the corresponding linear space. This simple idea is surprisingly powerful and has many famous applications.

This course provides a gentle introduction to Algebraic methods, illustrated by examples and focusing on basic ideas and connections to other areas. The topics covered in the class will include (but are not limited to):

Basic dimension arguments, Spaces of polynomials and tensor product methods, Eigenvalues of graphs and their application, the Combinatorial Nullstellensatz and the Chevalley-Warning theorem. Applications such as: Solution of Kakeya problem in finite fields, counterexample to Borsuk's conjecture, chromatic number of the unit distance graph of Euclidean space, explicit constructions of Ramsey graphs and many others.